**Université Ibn Khaldoun – Tiaret**
**Faculté des Sciences de la Matière/ Département de Physique**

**1$^{ère}$ Année Master Physique Médicale**
**Corrigé de l'examen de rattrapage: Introduction à la Physique des milieux ionisés**
**Année 2023/2024**

**Ex 01:** (5.5pt)

A) The mean free path is determined by

$$\lambda = \frac{1}{n\,\sigma}. \qquad (1pt)$$

The density of particles can be calculated from the equation of state p = n k T, so

$$\lambda = \frac{k\,T}{p\,\sigma}. \qquad (1pt)$$

So the final results for given pressures are:

a) $4 \times 10^{-6}$ m b) $4 \times 10^{-4}$ m c) $4 \times 10^{-2}$ m. (0.25pt)+ (0.25pt)+ (0.25pt)

B) The thermal velocity of ions is $v = \sqrt{\frac{3\,k\,T}{M}}$ (1pt). Mass of Xe ion is approximately $217.46 \times 10^{-27}$ kg. The time period between two subsequent collisions equals to the fraction of mean free path and thermal velocity:

$$\tau = \lambda\sqrt{\frac{m}{3\,k\,T}}. \qquad (1pt)$$

So the results are: a) $9.28 \times 10^{-8}$s b) $9.28 \times 10^{-7}$ s c) $9.28 \times 10^{-5}$s.

(0.25pt)+ (0.25pt)+ (0.25pt)

**Ex02:** (4,0pt)

1- En l'absence de champ magnétique et en régime permanent, l'équation

$$m_e\,\frac{D\mathbf{u}_e}{Dt} = -e\,(\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) - \nu_c\,m_e\,\mathbf{u}_e,$$

se réduit $-e\,\mathbf{E} - m_e\,\nu_c\,\mathbf{u}_e = 0$ (1,5pt)

La densité de courant électrique est définie comme suit

$J = -e n_e u_e$

En l'insérant dans l'équation de Langevin, on obtient l'expression de la densité de courant J

$$\mathbf{J} = \frac{n_e\, e^2}{m_e\, \nu_c}\, \mathbf{E} \qquad \text{(1,5pt)}$$

La loi d'Ohm stipule que

$$\mathbf{J} = \sigma_0\, \mathbf{E} \qquad \text{(0,5pt)}$$

La conductivité DC est donc la suivante

$$\sigma_0 = \frac{n_e\, e^2}{m_e\, \nu_c}. \qquad \text{(0,5pt)}$$

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**Ex 03:** (6,0pt)

(a) Since $n_e(r) = n_0 \exp\left(\frac{e\phi}{k_B T_e}\right)$ and $\frac{\partial n_e}{\partial r} \simeq -\frac{n_e}{\lambda}$,

$$\begin{aligned} \frac{\partial n_e}{\partial r} &= \frac{n_0 e}{k_B T_e}\frac{\partial \phi}{\partial r}\exp\left(\frac{e\phi}{kT_e}\right) \\ &\simeq -\frac{n_e}{\lambda} = -\frac{n_0}{\lambda}\exp\left(\frac{e\phi}{kT_e}\right) \; ; \end{aligned} \qquad \text{(1pt)}$$

$$\begin{aligned} \frac{e}{k_B T_e}\frac{\partial \phi}{\partial r} &= -\frac{1}{\lambda} \\ \frac{\partial \phi}{\partial r} &= -\frac{k_B T_e}{e\lambda} \; . \end{aligned} \qquad \text{(1pt)}$$

Hence,

$$\vec{E} = -\vec{\nabla}\phi = -\frac{\partial \phi}{\partial r}\,\vec{u}_r = \frac{k_B T_e}{e\lambda}\,\vec{u}_r \qquad \text{(1pt)}$$

Hence, the average Larmor radius is

$$\langle r_L \rangle = \frac{m\langle v_\perp \rangle}{eB} = \frac{mv_t}{eB} , \qquad \text{(1pt)}$$

or, equivalently,

$$v_t = \frac{eB\langle r_L \rangle}{m} .$$

On the other hand,

$$\begin{aligned} v_E = \frac{E}{B} &= \frac{k_B T_e}{e\lambda B} \qquad \text{(1pt)} \\ &= \frac{2k_B T_e}{m}\frac{m}{2e\lambda B} = v_t^2 \frac{m}{2e\lambda B} \\ &= v_t \underbrace{\frac{\langle r_L \rangle eB}{m}}_{v_t} \frac{m}{2e\lambda B} = \frac{v_t}{2\lambda}\langle r_L \rangle . \qquad \text{(1pt)} \end{aligned}$$

**Ex 04:** (04,5pt)

Consider a particle of charge q, initially at rest and placed at $x_i = -\frac{qE_0}{m\,\omega^2}$ that moves under the effect of a high-frequency electric field, $\vec{E} = E_0 \cos(\omega t)\overrightarrow{u_x}$

- Solve the equation of motion and describe the trajectory.

Solution:

(a) From the force equation, $\vec{F} = q\vec{E}$,

$$\begin{cases} m\ddot{x} = qE_0 \cos(\omega t) \\ \ddot{y} = 0 \\ \ddot{z} = 0 \end{cases} ; \quad \begin{cases} \frac{dv_x}{dt} = \frac{qE_0}{m}\cos(\omega t) & \text{(1pt)} \\ v_y(t) = v_y(t=0) = 0 & \text{(0,5pt)} \\ v_z(t) = v_z(t=0) = 0 & \text{(0,5pt)} \end{cases} .$$

Hence

$$v_x(t) = \int_0^t \frac{qE_0}{m}\cos(\omega\tau)\,d\tau = \frac{qE_0}{m\omega}\sin(\omega t) , \qquad \text{(1pt)}$$

$$x(t) = x_i + \int_0^t \frac{qE_0}{m\omega}\sin(\omega\tau)\,d\tau = \underbrace{\frac{-qE_0}{m\omega^2}}_{x_i} - \frac{qE_0}{m\omega^2}\cos(\omega t) + \frac{qE_0}{m\omega^2} ,$$

$$x(t) = -\frac{qE_0}{m\omega^2}\cos(\omega t) . \qquad \text{(1pt)}$$

The charge as an harmonic oscillatory motion around x = 0, with frequency ω and amplitude $\frac{qE_0}{m\ \omega^2}$ as shown in figure below

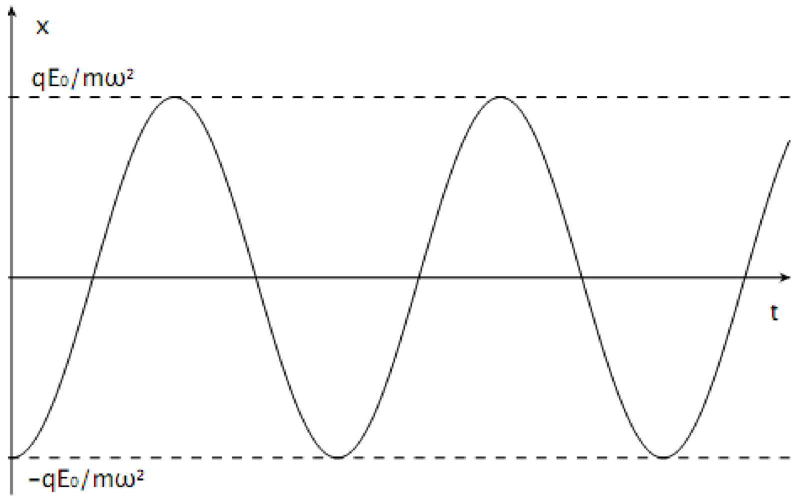


Figure: oscillatory trajectory (0,5pt)